NJL model in Four Dimension at Finite Temperature, Chemical Potential and Curvature

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Abstract

Two flavor Nambu-Jona-Lasinio model with N components is studied in curved space time at finite temperature and density in the leading 1/N expansion. In four space time dimension the model exhibits first order phase transition for positive curvature. Whereas in flat space an increase in temperature induces a second order phase transition even at finite density, in curved space the transition becomes first order. We obtain the phase boundary in density-temperature plane and exhibit its behavior with changing curvature. A three dimensional plot of the phase boundary in μ-T-R space is drawn.

PACS Nos : 11.30.Rd ; 11.30.Qc ; 05.70.Fh Keywords : Chiral symmetry, Phase transition, Curvature.

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1 Introduction

Spontaneous symmetry breaking (SSB) is one of the most important concepts of all unified gauge theories i.e the underlying gauge symmetry is larger than that of the vacuum. Of particular interest is the expectation that at high temperatures, symmetries that are spontaneously broken today are restored and that during the evolution, the universe passed through a series of phase transition evolving from a higher symmetric phase to a lower symmetric phase associated with the spontaneous breakdown of gauge or perhaps global symmetry. It is very instructive to investigate how the phase transition takes place in quantum field theory in the environment of the early universe where the temperature, density, primordial external electromagnetic field and external gravity may all play important role [1].

In the standard grand unified theories, the Higgs field plays a most fundamental role. It is an elementary scalar field with self coupling and has Yukawa couplings with all other fundamental fields-fermions and gauge bosons giving them masses through its non-vanishing vacuum expectation value. There also exist theories such as technicolour model of electro-weak theory [2] where the Higgs field may be considered as a composite field of some more fundamental fermion fields. QCD is an another example of quantum field theory which is invariant under chiral transformations at the lagrangian level in the absence of quark mass matrix. However, the dynamics of the QCD are expected to be such that chiral symmetry is dynamically broken with the vacuum state having a nonzero quark-antiquark condensate $\langle 0 \,|\bar{q}q|\, 0\rangle$ and the Goldstone theorem then requries the existence of approximately massless pseudo-scalar mesons in the hadron spectrum. To study chiral phase transition in QCD we need a non-perturbative treatment and a particularly attractive framework to study such systems is the Nambu-Jona -Lasinio (NJL) model [3]. Linear sigma model is another such model that is amenable to such a study. The NJL model (for a recent review see [4] and references therein) provides a useful framework for studying dynamical chiral symmetry breaking

non-perturbatively in the 1/N expansion, where starting from massless quarks, a vacuum condensate $\langle \bar{q}q \rangle \neq$ appears and a dynamical fermion mass is generated with the breakdown of chiral symmetry in the ground state. Further, NJL model can also be considered as a prototype of composite Higgs model and we will use it in the present study to investigate the breakdown of higher symmetry in the early universe.

The NJL model four fermion theory in flat space-time in arbitrary dimensions has been studied [5] using the 1/N expansion method. It is shown that chiral symmetry is restored in the theory under consideration for sufficiently high temperature or chemical potential. It is found that for space-time dimensions $2 \leq D < 3$ both a first order and second order phase transition occur depending on the value of the four fermion coupling while for $3 \leq D < 4$ only the second order phase transition exists. Recently an investigation of the chiral phase structure of the four fermion theory in curved space-time has been made [6-13]. In particular the existence of a first order phase transition induced by curvature from the chiral symmetric phase to the chiral non-symmetric one has been shown. Chiral symmetry structure with nontrivial topoligies [11, 13, 14] under the combined action of magnetic fields and curved space-time has also been studied [15]. In the present paper we will investigate the D=4 NJL model in curved space-time at finite temperature, density and curvature using the schwinger proper time method [16]. We will also take into account the effect of temperature and density on the leading contribution on curvature. Linear Sigma model will be topic of our subsequent study. The existence of first order phase transition at finite temperature and curvature is confirmed and the phase diagram in the curvature, temperature and density plane is obtained.

2 Four Fermion Theory in Curved Space-time

The Nambu Jona Lasinio model in curved space time is defined by the action

$$S = \int d^D x \sqrt{(-g)}\, [\, i\,\bar{\psi}\gamma^{\mu}(x) \bigtriangledown_{\mu} \psi + \frac{\lambda}{2N}(\, (\bar{\psi}\psi)^2 + (\bar{\psi} i\, \gamma_5\, \vec{\tau}\, \psi)^2)\,] \tag{1}$$

where g is the determinant of the space time metric ,$\gamma^{\mu}(x)$ the Dirac matrix in curved space-time, $\bigtriangledown_{\mu}\psi$ the covariant derivative of the fermion field ψ and N is the number of colours, we take the number of flavours to be two. We work in the scheme of the 1/N expansion and perform our calculations in the leading order of the expansion. For practical purposes it is more convenient to introduce the auxiliary fields σ and $\vec{\pi}$ and consider the equivalent action.

$$S = \int d^D x \sqrt{(-g)}\, [i\,\bar{\psi}\gamma^{\mu}(x) \bigtriangledown_{\mu} \psi - \frac{N}{2\lambda}(\sigma^2 + \pi^2) - \bar{\psi}(\sigma + i\,\gamma_5\, \vec{\tau}\, \vec{\pi})\psi] \tag{2}$$

Replacing σ and $\vec{\pi}$ by the solutions of the Euler-Lagrangian equations arising from (2) we reproduce the action (1). This is because the fields σ and $\vec{\pi}$ are not independent degrees of freedom in (2) and the Euler-Langrangion equations for σ and $\vec{\pi}$ are infact constraint equations which fix σ and $\vec{\pi}$ given ψ and $\bar{\psi}$.

If a non vanishing vacuum expectation value is assigned to the auxiliary field σ, then there appears a mass term for the fermion field ψ and the discrete chiral symmetry is eventually broken.

The effective potential (with N factored out) in the leading order of the 1/N expansion is then given by

$$V(\sigma, \pi) = \frac{1}{2\lambda}\,(\sigma^2 + \vec{\pi}^2) + i\,Tr\,ln\,\langle x \,|\, (i\gamma^{\mu}(x) \bigtriangledown_{\mu} - (\sigma + i\,\gamma_5\, \vec{\pi})\,|\, x\rangle \tag{3}$$

In equation (3) the variables σ and $\vec{\pi}$ are regarded as constant. Using the schwinger proper time method [16] we rewrite the second term on the right hand side of equation (4) as

$$V(\sigma, \pi) = \frac{1}{2\lambda}(\sigma^2 + \pi^2) + i\,Tr\,ln\,S\,(x, x; s)\Big|_{s=\sigma + i\gamma_5\,\vec{\pi}} \tag{4}$$

where the Green function S defined by

$$S(x,y;s) = \left\langle x \right| (i\gamma^{\mu}(x) \bigtriangledown_{\mu} - s)^{-1} \left| y \right\rangle \tag{5}$$

is the solution of the equation

$$(i\gamma^{\mu}(x) \bigtriangledown_{\mu} - s) S(x,y;s) = \frac{1}{\sqrt{(-g(x))}} \delta^{D}(x-y) \tag{6}$$

Thus the effective potential is described by the two point Green's function S(x,x;s) of the massive free fermion in curved spacetime. Using the Green function obtained in the approximation of keeping only linear terms in the curvature, Inagaki, Muta and Odintsov [8], obtained the effective potential which in D-dimensions read as follows:

$$\begin{aligned} V(\sigma,0) &= \frac{\sigma^2}{2\lambda} - iTr \int_0^{\sigma} ds \int \frac{d^D k}{(2\pi)^4} [(\gamma^a k_a + s) \frac{1}{k^2 - s^2} \\ & -\frac{R}{12}(\gamma^a k_a + s)\frac{1}{(k^2-s^2)^2} + \frac{2}{3} R_{\mu,\nu} k^{\mu} k^{\nu} (\gamma^a k_a + s) \\ & \times \frac{1}{(k^2-s^2)^3} - \frac{1}{2}\gamma^a J^{cd} R_{cda\mu} k^{\mu} \frac{1}{(k^2-s^2)^2}] \end{aligned} \tag{7}$$

where $J^{a\,b} = \frac{1}{4}[\gamma^a, \gamma^b]$ and latin indices are vierbein indices.

The effective potential is divergent in two and four dimensions and is finite in three dimensions in the leading 1/N expansion. The four fermion theory is renormalizable in 2-D flat space. Using the renormalization condition $\left.\frac{\partial^2 V_0(\sigma)}{\partial \sigma^2}\right|_{\sigma=M} = \frac{M^{D-2}}{\lambda_r}$ where M is the renormalization scale. The renormalised effective potential in D dimensions is given by [8]

$$\begin{aligned} \frac{V(\sigma,0)}{M^D} &= \frac{1}{2\lambda_r}\frac{\sigma^2}{M^2} + \frac{Tr\mathbf{1}}{2\,(4\pi)^{\frac{D}{2}}}\,(D-1)\,\Gamma\,(1-\frac{D}{2})\,\frac{\sigma^2}{M^2} \\ & -\frac{Tr\mathbf{1}}{(4\pi)^{\frac{D}{2}} D}\,\Gamma\,(1-\frac{D}{2})\,\frac{\sigma^D}{M^D} - \frac{Tr\mathbf{1}}{(4\pi)^{\frac{D}{2}}}\frac{R}{M^2}\frac{1}{24} \\ & \times \Gamma\,(1-\frac{D}{2})\,\frac{\sigma^{D-2}}{M^{D-2}} \end{aligned} \tag{8}$$

We now obtain the four dimensional limit of the NJL model in the MS renormalization scheme given by

$$\begin{aligned}\frac{V(\sigma,0)}{M^4} &= \frac{1}{2\lambda}\,(\frac{\sigma}{M})^2 - \frac{1}{4\,\pi^2}\,(1+3\;ln\,4\pi - 3\,\gamma)\,(\frac{\sigma}{M})^2 \\ &-\frac{1}{8\,\pi^2}\,(\,ln\,(\frac{\sigma}{M})^2 - \frac{3}{2} - ln\,4\pi + \gamma)\,(\frac{\sigma}{M})^4 \\ &-\frac{R}{48\,M^2\pi^2}\,(\;ln\,(\frac{\sigma}{M})^2 - 1 - ln\;4\pi + \gamma)\,(\frac{\sigma}{M})^2 \end{aligned} \tag{9}$$

Alternatively one could regularize the divergent part by cutting off the momentum integral at finite cutoff Λ [6]. This gives

$$\begin{aligned} V(\sigma,0) &= \frac{\sigma^2}{2\lambda} - \frac{1}{(4\pi)^2}\,[\;\sigma^2\Lambda^2 + \Lambda^4\,ln\,(1+\frac{\sigma^2}{\Lambda^2}) - \sigma^4\,ln\,(1+\frac{\Lambda^2}{\sigma^2})] \\ &-\frac{1}{(4\pi)^2}\,\frac{R}{6}\,[\;-\sigma^2\,ln\,(1+\frac{\Lambda^2}{\sigma^2}) + \frac{\Lambda^2\sigma^2}{\Lambda^2+\sigma^2}\;] \end{aligned} \tag{10}$$

The two expression can be shown to be equivalent after carrying out the renormalization of the coupling constant. The ground state of the theory is determined by the minimum of the effective potential (10) namely, by solving the gap equation

$$\frac{\partial V(\sigma,0)}{\partial\sigma}|_{\sigma=m} = 0 \tag{11}$$

For $\lambda_r > \lambda_c = \frac{2\pi^2}{1+3ln\,4\,\pi - 3\,\gamma}$, the minimum of the effective potential is located at the nonvanishing σ, the chiral symmetry is broken down dynamically and a dynamical fermion mass is generated. At the critical point the effective potential has two degenerate local minima obtained by putting

$$V(\sigma,0) = V(m,0) = 0 \tag{12}$$

The solution of the gap equation and the value of the critical curvature can be obtained numerically. First, we fix the coupling constant λ_r greater than the critical value λ_c corresponding to the broken symmetric phase. To study the phase structure in curved space-time we evaluate the effective potential (10) by varying the space-time curvature. We see from Fig.1a that the chiral symmetry is restored as R is increased for a fixed λ. The phase transition induced by curvature is of first

order as can be seen from Fig.1b. Similar results were obtained by the authors of ref. [7] who showed that a first order phase transition is induced by curvature in 3-dimensional as well. Phase structure of NJL model in curved space time with non-trivial topology has been discussed in the literature. It has been shown that the combined effect of topology and curvature on phase transition from chirally symmetric to chirally broken phase is such as to make the transition first order with the growth in curvature as compared to a second order phase transition at zero curvature [13].

3 Phase Structure at Finite Temperature and Chemical Potential with Curvature

Four fermion theories in flat space at finite temperature T and chemical potential μ in arbitrary dimension has been investigated [5] in the leading order of the 1/N expansion. The theory under consideration is renormalisable below four dimensions and gives an insight into the phase structure of the theory in four dimensions. Following the standard procedure of the Matsubara frequency sums [12] we calculate the effective potential in our theory in the leading order of the 1/N expansion by replacing the phase space integral in eqs (7) by

$$\int \frac{d^4k_e}{(2\pi)^4} \longrightarrow i\frac{1}{\beta}\sum_{n=0}^{\infty}\frac{d^3\mathbf{k_e}}{(2\pi)^3} \tag{13}$$

where the four momentum k^μ is given by

$$k^\mu = (iw_n - \mu, \mathbf{k}) \tag{14}$$

and the discrete variable w_n is given by $\frac{(2n+1)\pi}{\beta}$

$$\begin{aligned} V^\beta(\sigma,0) &= \frac{\sigma^2}{2\lambda} + i\int_0^\sigma sds\int\frac{d^4k}{(2\pi)^4}[\frac{1}{k^2-s^2} - \frac{R}{12}\frac{1}{(k^2-s^2)^2} \\ &\quad + \frac{R}{6}\frac{k^2}{(k^2-s^2)^3}] + \frac{i}{\beta}\sum_{n=0}^{\infty}\int_0^\sigma sds\frac{d^3\mathbf{k}}{(2\pi)^3}[\frac{1}{k^2-s^2} \end{aligned}$$

$$-\frac{R}{12}\frac{1}{(k^2-s^2)^2}+\frac{R}{6}\frac{k^2}{(k^2-s^2)^3}] \tag{15}$$

$$V^{\beta}(\sigma,0)=V_0(\sigma)+V_R(\sigma)+V_{\beta}(\sigma) \tag{16}$$

where

$$\begin{aligned} V_{\beta}(\sigma) &= \frac{i}{\beta}\sum_{n=0}^{\infty}\int_0^{\sigma} sds\frac{d^3\mathbf{k}}{(2\pi)^3}[\frac{1}{k^2-s^2}-\frac{R}{12}\frac{1}{(k^2-s^2)^2} \\ &+\frac{R}{6}\frac{k^2}{(k^2-s^2)^3}] \end{aligned} \tag{17}$$

Performing integration over s, we get

$$\begin{aligned} V_{\beta}(\sigma) &= \frac{i}{2\beta}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\sum_{n=-\infty}^{\infty}\ln[(w_n+i\mu)^2+E_k^2]+\frac{i}{24}\frac{R\sigma^2}{\beta}\int\frac{d^3\mathbf{k}}{(2\pi)^3} \\ &\sum_{n=-\infty}^{\infty}\frac{1}{[(w_n+i\mu)^2+E_k^2]^2}-\frac{i}{12}\frac{R}{\beta}\int\frac{d^3\mathbf{k}}{(2\pi)^3} \\ &\sum_{n=-\infty}^{\infty}\frac{1}{[(w_n+i\mu)^2+E_k^2]} \end{aligned} \tag{18}$$

where $E_k^2=\mathbf{k^2}+\mathbf{s^2}$ and throwing away the β independent terms and carrying out the summation over n by the standard techniques we get

$$\begin{aligned} V_{\beta}(\sigma) &= \frac{-2}{\beta\pi^2}\int k^2dk[ln(1+e^{-\beta[\sqrt{(k^2+\sigma^2)}-\mu]}+\mu\rightarrow-\mu] \\ &+\frac{R\sigma^2}{12\pi^2}\int\frac{k^2dk}{(k^2+\sigma^2)^{\frac{3}{2}}}(\frac{1}{1+e^{\beta[\sqrt{(k^2+\sigma^2)}-\mu]}}+\mu\leftrightarrow-\mu) \end{aligned} \tag{19}$$

The effective potential $V^{\beta}(\sigma,0)$ at finite temperature, density and curvature is then given by

$$\begin{aligned} V^{\beta}(\sigma,0) &= \frac{\sigma^2}{2\,\lambda}-\frac{1}{4\pi^2}\,(\,1+3\,ln\,4\pi-3\,\gamma)\,\sigma^2 \\ &-\frac{1}{8\pi^2}(\,ln\frac{\sigma^2}{M^2}-\frac{3}{2}-ln\,4\pi+\gamma\,)\sigma^4 \\ &-\frac{R}{48\pi^2}(\,ln\,\frac{\sigma^2}{M^2}-1-ln\,4\pi+\gamma)\sigma^2 \end{aligned}$$

$$\frac{-2}{\beta\pi^2}\int k^2 dk[ln(1+e^{-\beta[\sqrt{(k^2+\sigma^2)}-\mu]}+\mu\rightarrow-\mu]$$
$$+\frac{R\sigma^2}{12\pi^2}\int\frac{k^2dk}{(k^2+\sigma^2)^{\frac{3}{2}}}(\frac{1}{1+e^{\beta[\sqrt{(k^2+\sigma^2)}-\mu]}}+\mu\leftrightarrow-\mu)$$
$$+\frac{R\sigma^2\beta}{6\pi^2}\int\frac{k^2dk}{(k^2+\sigma^2)}(\frac{e^{\beta[\sqrt{(k^2+\sigma^2}-\mu)]}}{(1+e^{\beta[\sqrt{(k^2+\sigma^2)}-\mu]})^2}\mu\leftrightarrow-\mu) \tag{20}$$

At high densities and low temperatures, the matter is degenerate and the fermion distribution function can be approximated by the step function. The integral in (20) can be carried out analytically and we obtain

$$\begin{aligned}\frac{V^\beta(\sigma,0)}{M^4} \quad = \quad & \frac{\sigma^2}{2\lambda M^2}-\frac{1}{4\pi^2}\,(1+3\,ln\,4\pi-3\,\gamma)\,\frac{\sigma^2}{M^2}\\ & -\frac{1}{8\pi^2}\,(ln\,\frac{\sigma^2}{M^2}-\frac{3}{2}-ln\,4\pi+\gamma)\,\frac{\sigma^4}{M^4}\\ & -\frac{R}{48M^2\pi^2}\,(ln\,\frac{\sigma^2}{M^2}-1-ln\,4\pi+\gamma)\,\frac{\sigma^2}{M^2}\\ & -\frac{2}{3\pi^2}\,\frac{1}{M^4}\,[\,\frac{\mu}{4}\,(\mu^2-\sigma^2)^{\frac{3}{2}}-\frac{3}{8}\sigma^2\,\mu\,(\mu^2-\sigma^2)^{\frac{1}{2}}\\ & +\frac{3}{8}\,\sigma^4\,ln\,(\frac{\mu+\sqrt{(\mu^2-\sigma^2)}}{\sigma})]+\frac{R\sigma^2}{12M^4\pi^2}\,[-\frac{\sqrt{(\mu^2-\sigma^2)}}{\mu}\\ & +ln\,(\frac{\mu+\sqrt{(\mu^2-\sigma^2)}}{\sigma})\,]\end{aligned} \tag{21}$$

But at high temperatures and low densities relevant to the study of phase transition in the early universe the integrals in eqs (20) can be done analytically by the usual tecniques [17] and we obtain

$$\begin{aligned}\frac{V^\beta(\sigma,0)}{M^4} \quad = \quad & \frac{1}{2\lambda}(\frac{\sigma}{M})^2-\frac{1}{4\pi^2}\,(1+3\,ln\,4\pi-3\,\gamma)\,\frac{\sigma^2}{M^2}\\ & -\frac{1}{8\pi^2}\,(ln\,\frac{\sigma^2}{M^2}-\frac{3}{2}-ln\,4\pi+\gamma)\,\frac{\sigma^4}{M^4}\\ & -\frac{R}{48M^2\,\pi^2}\,(ln\,\frac{\sigma^2}{M^2}-1-ln\,4\pi+\gamma)\,\frac{\sigma^2}{M^2}\\ & -\frac{-2}{\pi^2\beta^4\,M^4}[\frac{7}{360}\pi^4-\frac{\sigma^2\beta^2\pi^2}{24}+\frac{\sigma^4\beta^4}{32}ln\sigma^2\beta^2+\frac{\sigma^4\beta^4}{32}c]\end{aligned}$$

$$+\frac{R\sigma^2}{48\pi^2}(ln\frac{\beta\sigma}{\pi}+\gamma) \tag{22}$$

where c=$\frac{3}{2}+2\ ln4\ \pi-2\gamma$. We plot the effective potential(20) as a function of σ for different values of R at fixed temperature and density. We observe from Fig.2 that at finite temperature symmetry is restored at lower values of curvature as compared to the zero temperature case. The solution of the gap equation corresponds to the dynamical mass of the fermion and is plotted in Fig.3a at zero chemical potential as a function of curvature for different values of temperature. We observe that with the increase in temperature, chiral symmetry is restored at lower values of curvature. In Fig.3b, we show that solution of the gap equation as a function of chemical potential for different values of curvature at T=0. We again find that the symmetry is restored at lower density (lower chemical potential) as the curvature increases.

4 Conclusion

In this paper we have investigated the phase structure of NJL model in curved space time using 1/N expansion and working in the linear curvature approximation. But in one discussion we considered rather large values of curvature and it would appear that the results obtained may not be reliable. However, it has been pointed out [8] that since terms quadratic and/or higher in R are not divergent, R^2 terms are expected to be relatively suppressed compared to linear terms. It has also been demonstrated in de-sitter space [12, 11] and Einstein universe [10] that the results obtained in linear approximation are indeed the exact results. A distinguishing feature of our investigation is that in the presence of external gravity with positive R, the chiral symmetry restoring phase transition is first order even as in flat space-time, the transition is second order with temperature and becomes first order in the presence of density. In Fig.4 we have shown the phase boundary in temperature, chemical potential plane for different values of curvature. We notice that with the increase in R, phase transition takes place at

lower temperature and density. In Fig.5 we have shown the phase boundary curve in temperature, chemical potential plane.

Acknowledgements

We would like to thank Professor J.V. Narlikar for providing hospitality at the Inter-University Centre for Astronomy and Astrophysics, Pune 411 007, India where this work was initiated.

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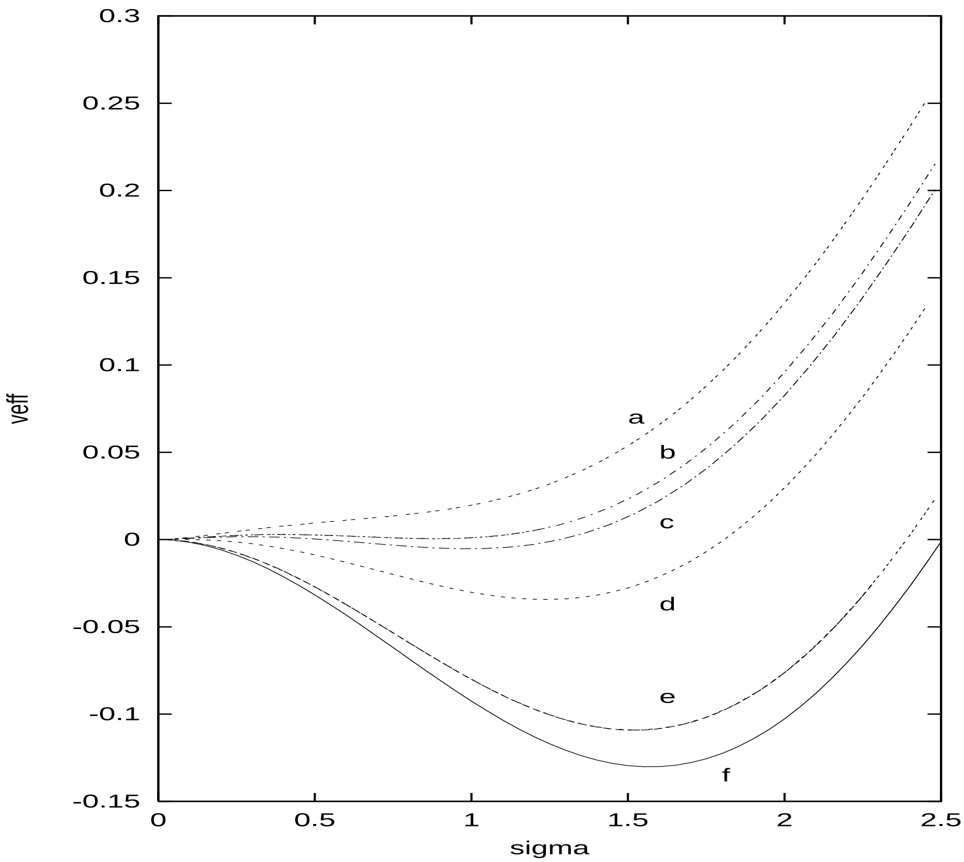

Figure 1: Behaviour of Effective Potential $\frac{V}{M^4}$ as a function of $\frac{\sigma}{M}$ for different values of the curvature for $\lambda > \lambda_{cr}(\lambda = 10)$. The curves a, b, c, d, e and f are for R = 16, 13, 12, 8, 0 and -2 respectively.

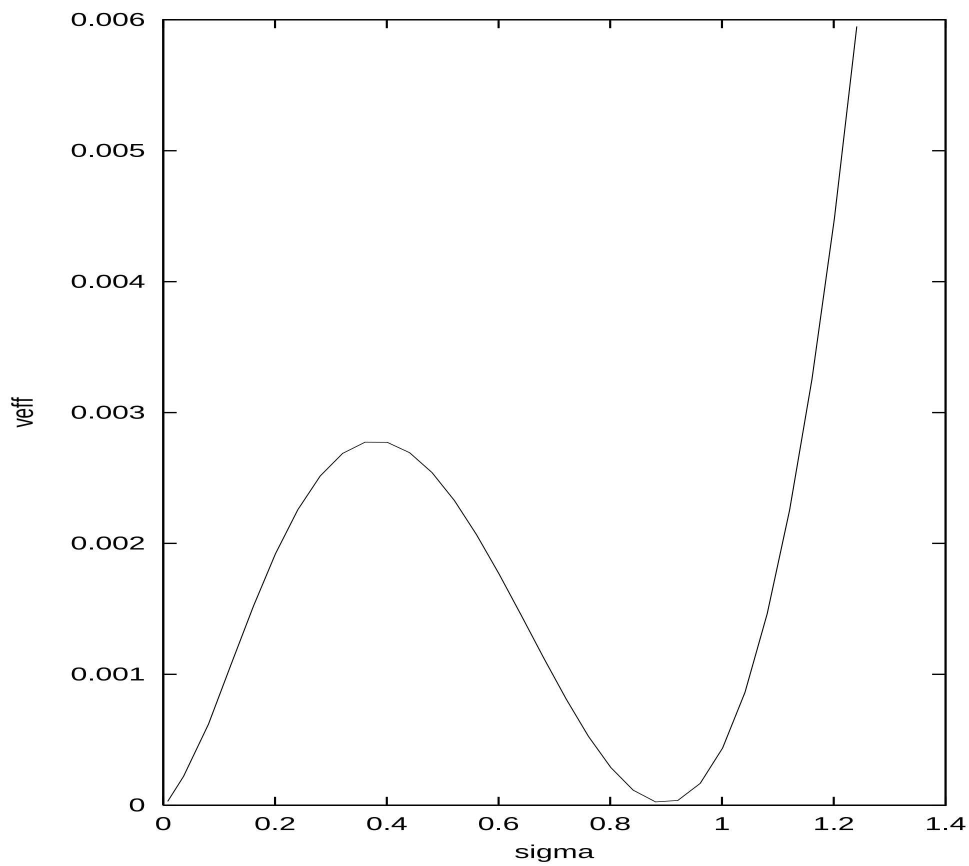

Figure 2: Effective potential $\frac{V}{M^4}$ for R$/M^2$ =12.91 showing the first order phase transition.

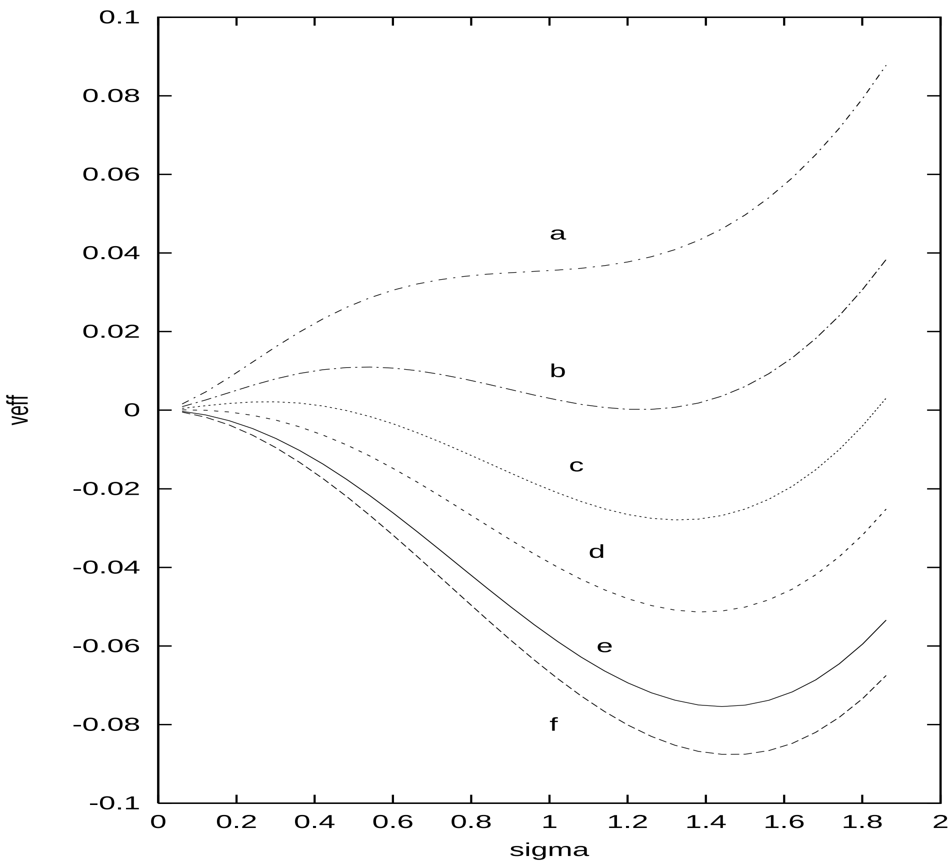

Figure 3: Behaviour of effective potential as a function of σ/M for different values of the curvature at fixed temperature T/M =0.5. The curves a, b, c, d, e and f are for R/M = 10, 6.5, 4, 2, 0, -1 respectively.

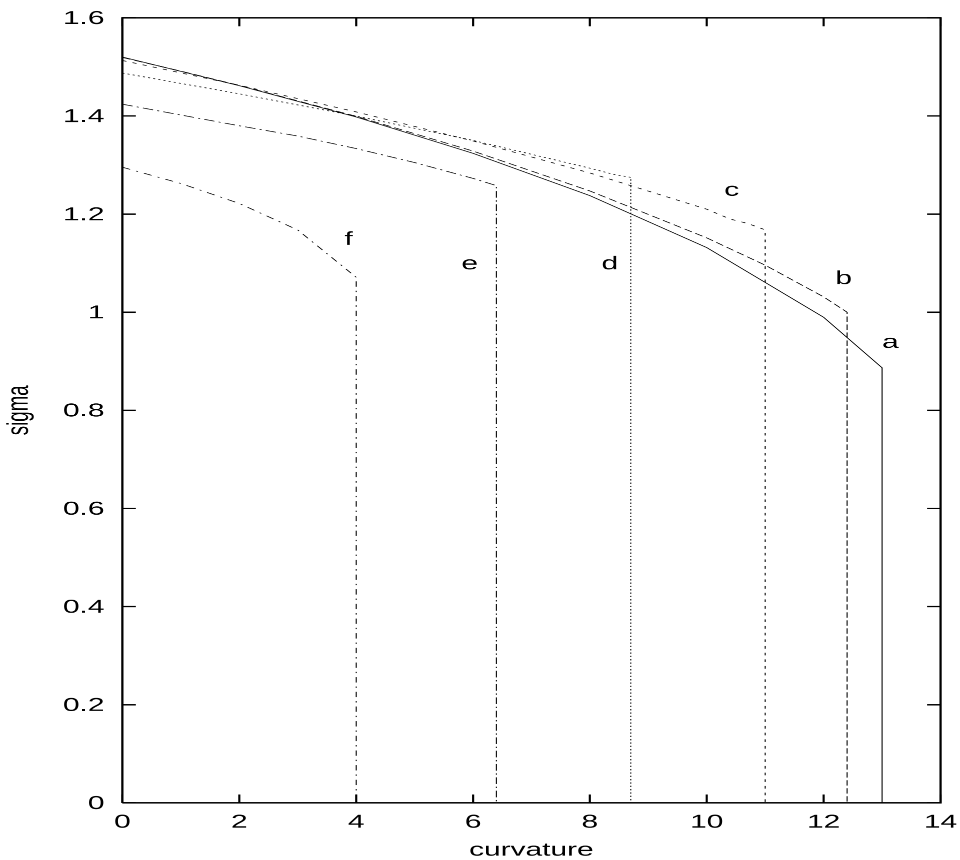

Figure 4: Sigma vs curvature for different values of temperature. The curves a, b, c, d, e and f are for $\frac{T}{M} = 0$, 0.2, 0.3, 0.4, 0.5, and 0.6 respectively.

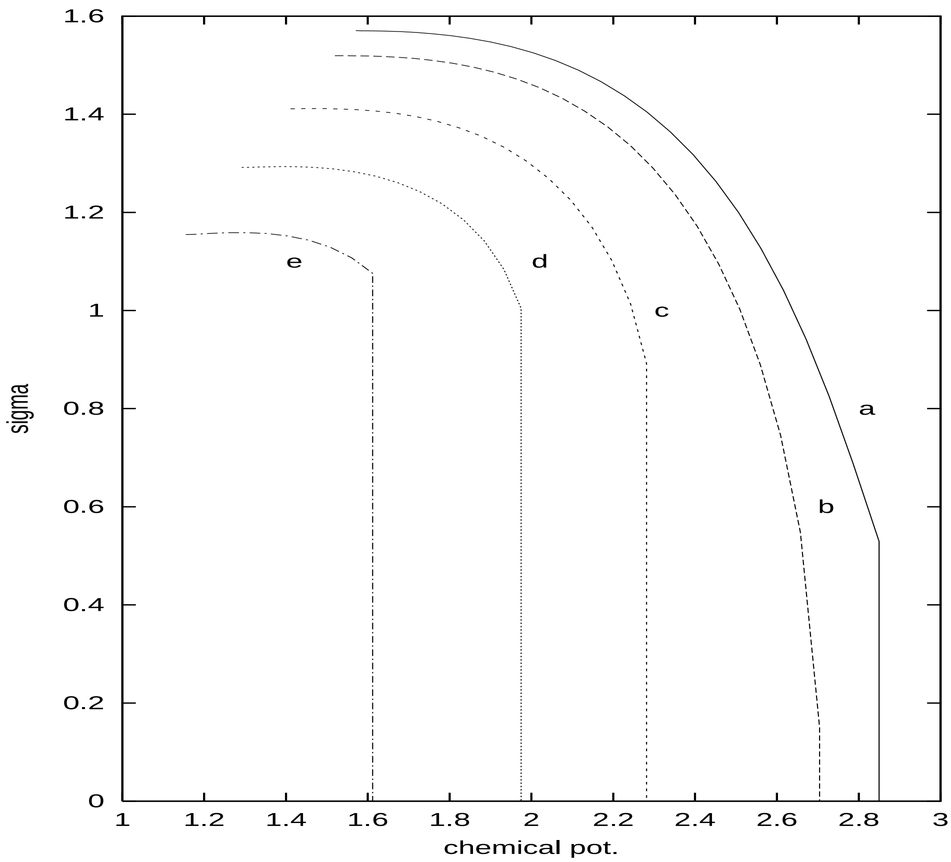

Figure 5: Sigma vs chemical potential for different values of curvature. The curves a, b, c, d and e are for R/M^2 = 0, 2, 4, 6, 8 and 10 respectively.

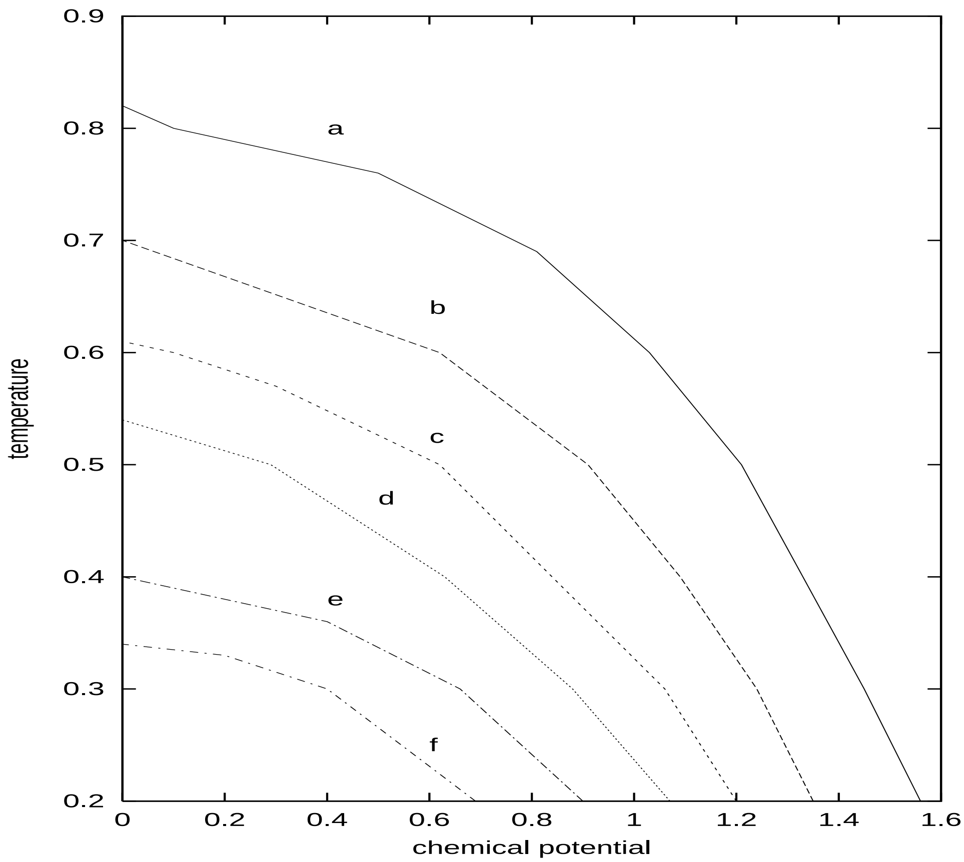

Figure 6: Temperature vs chemical pot. for different values of curvature. The curves a, b, c, d, e and f are for R/M^2 = 0, 2, 4, 6, 8 and 10 respectively.

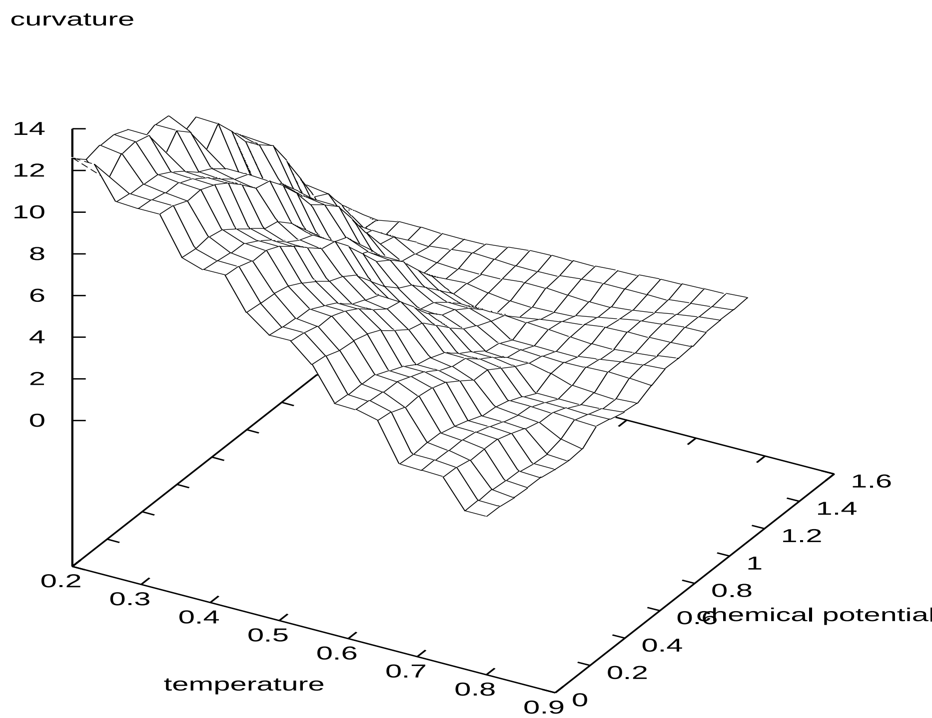

Figure 7: Phase diagram in the temperature, curvature and chemical potential plane.